Total No. of Questions : 9]

SEAT No. :

P1477

[Total No. of Pages : 5

[6002]-104

S.E. (Civil)

ENGINEERING MATHEMATICS-III

(2019 Pattern) (Semester-III) (207001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Questions No. 1 is compulsory.***
2) ***Answer Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.***
3) ***Non- programmable electronic packet calculator is allowed.***
4) ***Figures to the right indicate full marks.***
5) ***Assume Suitable data if necessary.***
6) ***Neat diagrams must be drawn wherever necessary.***

Q1) Attempt the following.

a) The first four moments of distribution about mean one 0, 16, –64 and 162, then standard deviation of a distribution is____. **[2]**

i) 21 ii) 12

iii) 16 iv) 4

b) The value of ∇_r^2 is_____ **[2]**

i) $-\frac{2}{r}$ ii) $\frac{2}{r}$

iii) $\frac{1}{r}$ iv) 0

c) For $\overline{F}=3x^2\hat{i}+(2xz-y)\hat{j}+z\hat{k}$, the value of $\int \overline{F}.d\overline{r}$ along straight line joining points (0,0,0) and (2,1,3) is _____. **[2]**

i) 15 ii) 14

iii) 16 iv) 8

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d) The most general solution of PDE $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ is ____. **[2]**

i) $u(x,t) = (c_1 \cos mx + c_2 \sin mx)\, e^{-m^2 t}$

ii) $u(x,t) = (c_1 \cos mx + c_2 \sin mx)\, (c_3 \cos mt + c_4 \sin mt)$

iii) $u(x,y) = (c_1 \cos mx + c_2 \sin mx)\, (c_3 e^{my} + c_4 e^{-my})$

iv) $u(x,y) = (c_1 e^{mx} + c_2 e^{-mx})\, (c_3 \cos my + c_4 \sin my)$

e) A throw is made with two dice. The probability getting a score of 10 is _____. **[1]**

i) $\frac{1}{12}$

ii) $\frac{1}{6}$

iii) $\frac{1}{5}$

iv) $\frac{2}{3}$

f) The cross product of $\overline{a}$ & $\overline{b}$ is defined as $\overline{a} \times \overline{b} =$ **[1]**

i) $ab\cos\theta$

ii) $ab\sin\theta\, \hat{n}$

iii) $ab\sin\theta$

iv) $ab\cos\theta\, \hat{b}$

Q2) a) Calculate the first four moments about mean of the given distribution also find β_1 & β_2. **[5]**

x	2	2.5	3	3.5	4	4.5	5
f	5	38	65	92	70	40	10

b) Find coefficient of correlation from given data. **[5]**

$n = 25, \Sigma x = 75, \Sigma y = 100, \Sigma x^2 = 250, \Sigma y^2 = 500, \Sigma xy = 325.$

c) An unbiased coin is thrown 10 times. Find probability of getting. **[5]**

i) Exactly 6 heads

ii) At least 6 heads

OR

Q3) a) Find lines of regression for the following data. **[5]**

x	10	14	19	26	30	34	39
y	12	16	18	26	29	35	38

b) One percent of articles from a certain machine are defective. What is the probability of **[5]**

i) No defective

ii) One defective

c) Assuming that the diagram of 1000 brass plugs taken consecutively from machine form a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm. How many of the plugs are likely to be approved if the acceptable diagram is 0.752 ± 0.004 cm. **[5]**

[Given A (2.25) = 0.4878, A (+1.75) = 0.4599]

Q4) a) Find the angle between velocity and acceleration vectors

at $t = 0$ for $\vec{r} = e^{-t}\hat{i} + \log(t^2+1)\hat{j} - \tan t\,\hat{k}$. **[5]**

b) In what direction from the point (1, 0, 1) is the directional derivative of $\phi = x^2 y z^3$ a maximum? What is the magnitude of this maximum? **[5]**

c) Show that $\vec{F} = (2xz^3 + 6y)\,\hat{i} + (6x - 2yz)\,\hat{j} + (3x^2z^2 - y^2)\hat{k}$ is irrotational. Find scalar ϕ such that $\vec{F} = \nabla\phi$. **[5]**

OR

Q5) a) If directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at (1,1,1) has maximum magnitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. Hence find the values of *a, b, c*. **[5]**

b) Attempt any one **[5]**

i) $\nabla^2\left(\nabla.\frac{\vec{r}}{r^2}\right) = \frac{2}{r^4}$

ii) $\nabla^2\left(\frac{\vec{a}.\vec{b}}{r}\right) = 0$

c) Show that $\vec{F} = \frac{1}{r}\left[r^2\vec{a} + (\vec{a}.\vec{r})\vec{r}\right]$ is irrotational. **[5]**

Q6) a) Evaluate $\oint_c \overline{F}.d\,\overline{r}$ where $\overline{F}=(x^2+xy)\hat{i}+(x^2+y^2)\,\hat{j}$ where C is the square formed by $y = \pm 1$ and $x = \pm 1$, $\overline{r} = x\hat{i}+y\hat{j}+z\hat{k}$. **[5]**

b) Evaluate $\iint_s \overline{f}.\hat{n}ds$ where $\overline{f} = yz\,\hat{i}+zx\,\hat{j}+xy\,\hat{k}$ and S is the sphere $x^2+y^2+z^2=1$ in the first octant. **[5]**

c) Apply Gauss divergence theorem to evaluate $\iint_s \overline{f}.\hat{n}ds$ where $\overline{f} = 4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$, S being the closed cylinder $x^2 + y^2 = 4$ bounded by $z = 0$ and $z = 3$. **[5]**

OR

Q7) a) Using Green's lemma for $\overline{F}=(3x^2-8y^2)\hat{i}+(4y-xy)\,\hat{j}$ and the curve C bounding the region R formed by $x = 0$, $y = 0$ and $x + y = 1$, evaluate $\iint_R (\nabla\times\overline{F}).dxdy\,\hat{k}$. **[5]**

b) Using Gauss divergence theorem evaluate $\iint_s (\overline{F}.\overline{n})ds$ where $\overline{F}=x^2z\hat{i}+y\hat{j}-xz^2\hat{k}$ where S is the boundary of the region bounded by the surfaces $z = x^2 + y^2$ and $z = 4y$. **[5]**

c) A liquid mass is rotating with a constant angular velocity ω about a vertical axis (positive z-axis) under the action of gravity. Find the pressure at any point of the liquid, if the motion is steady. Use the equation $\frac{\partial\overline{q}}{\partial t}+(\overline{q}.\nabla)\overline{q}=\overline{F}-\frac{1}{\rho}\nabla p$ assigning the symbols appropriate meanings. **[5]**

Q8) a) If $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ represents the vibrations of the string of length fixed at both ends. Find the solution it. **[8]**

i) $y\,(0,t) = 0$

ii) $y\,(l,\,t) = 0$

iii) $\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0$

iv) $y\,(x,0) = l\,x - x^2 \quad 0 < x < l$

b) Solve the following one-dimensional heat flow equation, $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to conditions. **[7]**

i) $u(0,t) = 0, \forall t$

ii) $u(l,t) = 0 \ \forall t$

iii) $u(x,0) = x \quad 0 < x < l$

iv) $u(x, t)$ is bounded.

OR

Q9) a) If the wave equation of vibration of string is given by, $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. Find the solution $y(x, t)$, if, **[8]**

i) $y(0,t) = 0 \ \forall t$

ii) $y(l,t) = 0 \ \forall t$

iii) $y(x,0) = 0 \ \forall x$

iv) $$\left.\frac{\partial y}{\partial t}\right|_{t=0} = \begin{cases} ax & 0 < x < l/2 \\ a(l-x) & \frac{l}{2} < x < l \end{cases}$$

b) Solve, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if, **[7]**

i) u is finite for all t

ii) $u(0, t) = 0$

iii) $u(\pi, t) = 0$,

iv) $u(x, 0) = \pi x - x^2 \quad 0 \le x \le \pi$

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